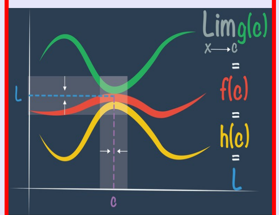
Calculus I

Lecture 22



Feb 19-8:47 AM

Class QZ 17

Given $f(x) = \dfrac{\sqrt{x^2+1}}{x+1}$

1) Find $\lim\limits_{x\to\infty} f(x)$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \frac{\infty}{\infty} \text{ I.F.}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\frac{\sqrt{x^2+1}}{x}}{\frac{x+1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}+\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x^2}\to 0}}{1+\frac{1}{x}\to 0}$$

$$= \frac{\sqrt{1}}{1} = \boxed{1}$$

2) Find $\lim\limits_{x\to-\infty} f(x)$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x+1} = \frac{\infty}{-\infty} \text{ I.F.}$$

$\vdots$

$$\lim_{x\to-\infty} \frac{-\sqrt{\frac{x^2+1}{x^2}}}{\frac{x+1}{x}}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}$$

$\vdots$

$$= \boxed{-1}$$

Jul 23-7:41 AM

$f(x) = \dfrac{\sqrt{x^2+1}}{x+1}$ $f(1) = \dfrac{\sqrt{1^2+1}}{1+1} = \dfrac{\sqrt{2}}{2}$ $f(2) = \dfrac{\sqrt{5}}{3}$

1) Domain $(-\infty,-1)\cup(-1,\infty)$ 2) V.A. $x=-1$

3) H.A. $y=1$ $(x\to\infty)$
$y=-1$ $(x\to-\infty)$

4) Y-Int $(0,1)$
x-Int None

Given $f'(x) = \dfrac{x-1}{(x+1)^2\sqrt{x^2+1}}$, $f''(x) = \dfrac{-2x^3+3x^2+3}{(x+1)^3\sqrt{(x^2+1)^3}}$

5) C.P. $f'(x)=0$ or und.

$x-1=0$
$x=1$
$\approx(1,.7)$

$(x+1)^2=0$
$x=-1$
Not in the domain

6) P.I.P.

$f''(x)=0$ or und.

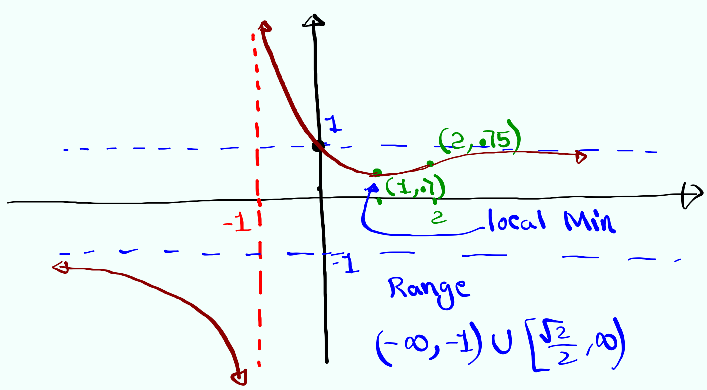
$-2x^3+3x^2+3=0$
⋮
$x\approx2$
$(2,.75)$

$(x+1)^3=0$
$x=-1$
Not in the domain

Jul 23-8:14 AM

7) Sign Chart

x	$-\infty$	-1		1		2	∞
$f'(x)$	$-$	○	$-$	●	$+$		$+$
$f''(x)$	$-$	○	$+$		$+$	●	$-$
$f(x)$							



Range
$(-\infty,-1)\cup\left[\frac{\sqrt{2}}{2},\infty\right)$

Jul 23-8:23 AM

Find x & y such that $x+y=20$ and x^2+y^2 is as small as possible. $\rightarrow y=20-x$

$x^2+(20-x)^2$ Minimize

$f(x)=x^2+(20-x)^2$ $\quad f'(x)$ $\quad f''(x)$

$f'(x)=2x+2(20-x)^1\cdot(-1)$

$f'(x)=2x-40+2x$

$f'(x)=4x-40$ $\qquad f''(x)=4>0$

CU

Min.

$f'(x)=0$

$4x-40=0$

$\boxed{x=10}$

$y=20-x$

$y=20-10 \rightarrow \boxed{y=10}$

Two numbers are 10 & 10

$x^2+y^2=10^2+10^2=200$ Smallest Value.

Jul 23-8:35 AM

A cylindrical can with no top has a volume of V cm^3 of water.

h, r

$V=\pi r^2 h$

$h=\dfrac{V}{\pi r^2}$

Find dimensions that will minimize the cost of making such can.

Materials needes Base + Side

Circle πr^2

Rectangula (once cut) $2\pi r h$

h

$2\pi r$

$$\text{Materials needed} = \pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$$

$$= \pi r^2 + \frac{2V}{r}$$

$$f(r)=\pi r^2+\frac{2V}{r}$$

$$f'(r)=2\pi r-\frac{2V}{r^2} \qquad f''(r)=2\pi+\frac{4V}{r^3}$$

$f'(r)=0$

$2\pi r^3-2V=0$

$r^3=\dfrac{2V}{2\pi}$

$\boxed{r=\sqrt[3]{\dfrac{V}{\pi}}}$

$$f''\left(\sqrt[3]{\frac{V}{\pi}}\right)=2\pi+\frac{4V}{\left(\sqrt[3]{\frac{V}{\pi}}\right)^3}>0$$

CU

Min

$f'(x)=0$

$r=\sqrt[3]{\dfrac{V}{\pi}}$

$V=\pi r^2 h$

$V=\pi\left(\sqrt[3]{\dfrac{V}{\pi}}\right)^2 h$

$V=\pi\sqrt[3]{\dfrac{V^2}{\pi^2}}\,h \rightarrow \cancel{V^3}=\cancel{\pi^3}\cdot\dfrac{\cancel{V^2}}{\cancel{\pi^2}}h^3$

$V=\pi h^3$

$h^3=\dfrac{V}{\pi}$

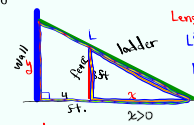
$\boxed{h=\sqrt[3]{\dfrac{V}{\pi}}}$

Jul 23-8:42 AM

A fence is 8 ft tall.

It runs parallel to a wall 4 ft away.

Find the smallest ladder (Smallest length) that will reach the ground over the fence to the wall.



Length of the ladder

$L^2=(x+4)^2+y^2$

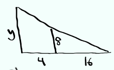
$L=\sqrt{(x+4)^2+y^2}$

$L=\sqrt{(16+4)^2+10^2}=\sqrt{500}$ $\approx \boxed{22.4 \text{ ft}}$

Need to minimize $(x+4)^2+y^2$

$\frac{8}{x}=\frac{y}{4+x}$ $\quad xy=8(4+x)$ $\quad y=\frac{8(4+x)}{x}$

Minimize



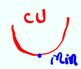
$f(x)=(x+4)^2+\left(\frac{8(4+x)}{x}\right)^2$

$f(x)=(x+4)^2+\frac{64(16+8x+x^2)}{x^2}$ $\qquad \frac{8}{16}=\frac{y}{20}$

$f(x)=(x+4)^2+\frac{1024}{x^2}+\frac{512}{x}+64$ $\qquad \frac{1}{2}=\frac{y}{20}$ $\quad \boxed{y=10}$

$f'(x)=2(x+4)\cdot 1-\frac{2048}{x^3}-\frac{512}{x^2}$

$f''(x)=2+\frac{6144}{x^4}+\frac{1024}{x^3}>0$



$f'(x)=0$

$2x+8-\frac{2048}{x^3}-\frac{512}{x^2}=0$

Multiply by x^3

$2x^4+8x^3-2048-512x=0$

$2x^4+8x^3-512x-2048=0$

$x^4+4x^3-256x-1024=0$

$$\begin{array}{r|rrrr} 16 & 1 & 4 & -256 & -1024 \\ & & 16 & 320 & 1024 \\ \hline & 1 & 20 & 64 & 0 \end{array}$$

$\boxed{x=16}$

Jul 23-9:02 AM

1) Find $\int \sqrt{x}\,dx = \qquad +C$ Indefenite Integral

$$\int x^{1/2}dx=\frac{x^{1/2+1}}{1/2+1}+C=\frac{x^{3/2}}{3/2}+C=\boxed{\frac{2}{3}x\sqrt{x}+C}$$

2) Evaluate $\int_1^9 \sqrt{x}\,dx$ # Defenite Integral

$$=\frac{2}{3}x\sqrt{x}\Big|_1^9=\frac{2}{3}\left[9\sqrt{9}-1\sqrt{1}\right]=\frac{2}{3}[27-1]$$

$$=\frac{2}{3}\cdot 26$$

$$=\boxed{\frac{52}{3}}$$

Jul 23-9:51 AM

Recall from Algebra

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$\frac{1}{x^n} = x^{-n}$

1) Find $\int \frac{1}{\sqrt[3]{x^2}}\,dx$

$= \int x^{-2/3}\,dx = \frac{x^{-2/3+1}}{-2/3+1} + C = \frac{x^{1/3}}{1/3} + C = \boxed{3\sqrt[3]{x} + C}$

2) Evaluate $\int_1^8 \frac{1}{\sqrt[3]{x^2}}\,dx$

$= 3\sqrt[3]{x}\Big|_1^8 = 3\left(\sqrt[3]{8} - \sqrt[3]{1}\right) = 3(2-1)$

$= \boxed{3}$

Jul 23-9:56 AM

what is wrong?

$\int_{-1}^{2} \frac{4}{x^3}\,dx = \frac{-2}{x^2}\Big|_{-1}^{2} = \frac{3}{2}$

Not cont. at $x=0$

0 is in $[-1, 2]$.

$\int \frac{4}{x^3}\,dx = \int 4x^{-3}\,dx = 4\cdot\frac{x^{-3+1}}{-3+1} + C$

$= 4\frac{x^{-2}}{-2} + C$

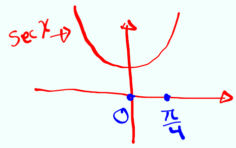
$= \frac{-2}{x^2} + C$

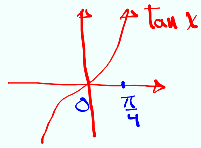
$\frac{-2}{x^2}\Big|_{-1}^{2} = -2\left[\frac{1}{2^2} - \frac{1}{(-1)^2}\right] = -2\left[\frac{1}{4} - 1\right] = -2\cdot\frac{1-4}{4}$

$= \frac{-2(-3)}{4} = \frac{3}{2}$

Jul 23-10:00 AM

Evaluate $\displaystyle\int_0^{\pi/4} \overbrace{\sec x \tan x}^{\text{Cont.}}\, dx = \sec x \Big|_0^{\pi/4}$





$= \sec\frac{\pi}{4} - \sec 0$

$= \boxed{\sqrt{2} - 1}$

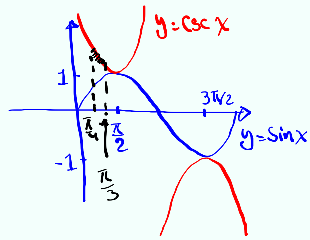
Evaluate $\displaystyle\int_{\pi/4}^{\pi/3} \csc^2 x\, dx = -\cot x \Big|_{\pi/4}^{\pi/3}$

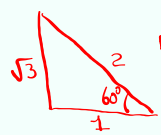
$\csc x = \dfrac{1}{\sin x}$

$= -\left[\cot \frac{\pi}{3} - \cot \frac{\pi}{4}\right]$ 60° 45°

$= -\left[\frac{1}{\sqrt{3}} - 1\right] = 1 - \frac{1}{\sqrt{3}}$

$= \boxed{1 - \frac{\sqrt{3}}{3}}$



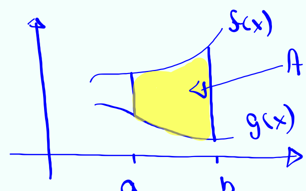


$\tan 60° = \sqrt{3}$

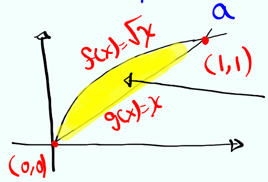
$\cot 60° = \frac{1}{\sqrt{3}}$

Jul 23-10:08 AM

If $f(x) \geq g(x)$ for $a \leq x \leq b$ and $f(x)$ & $g(x)$ are continuous on $[a,b]$, then the area bounded by $f(x)$, and $g(x)$ on $[a,b]$ is



$A = \displaystyle\int_a^b [f(x) - g(x)]\, dx$



$A = \displaystyle\int_0^1 [\sqrt{x} - x]\, dx$

$= \left(\dfrac{x^{3/2}}{3/2} - \dfrac{x^2}{2}\right)\Big|_0^1$

$= \left(\frac{2}{3} x\sqrt{x} - \frac{1}{2}x^2\right)\Big|_0^1$

$= \frac{2}{3}\cdot 1\sqrt{1} - \frac{1}{2}\cdot 1^2 - 0$

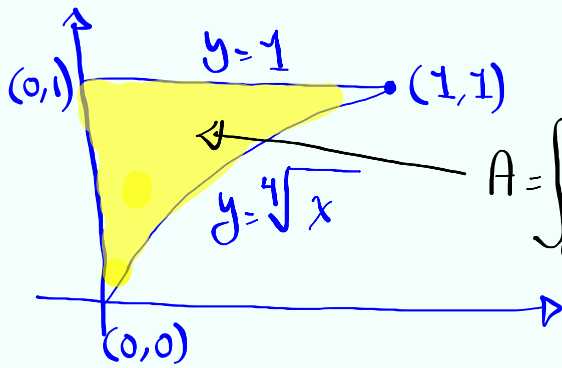
$\frac{2}{3} - \frac{1}{2} =$

$\frac{4-3}{6} =$

$= \boxed{\frac{1}{6}}$

Jul 23-10:18 AM

Find the shaded area



$x^{\frac{1}{4}}$

$$A=\int_0^1\left[1-\sqrt[4]{x}\right]dx$$

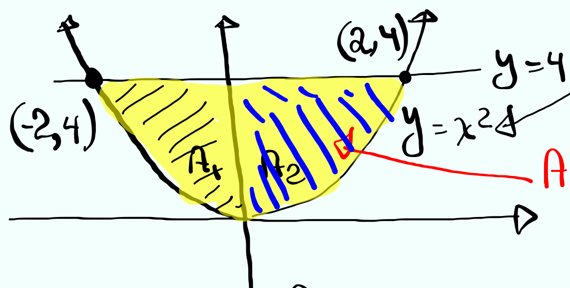
$$=\left[x-\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1}\right]\Bigg|_0^1$$

$$=\left(x-\frac{4}{5}x^{5/4}\right)\Bigg|_0^1=1-\frac{4}{5}\cdot 1^{5/4}-0=1-\frac{4}{5}=\boxed{\frac{1}{5}}$$

Jul 23-10:25 AM

Find the shaded area

even function



$$A=\int_{-2}^{2}\left[4-x^2\right]dx$$

$$A=2\int_0^2\left[4-x^2\right]dx=2\left[\left(4x-\frac{x^3}{3}\right)\right]\Bigg|_0^2$$

$$=2\left[\left(4\cdot 2-\frac{2^3}{3}\right)-(0)\right]=2\left[8-\frac{8}{3}\right]$$

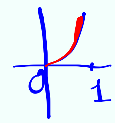
$$=2\cdot\frac{16}{3}=\boxed{\frac{32}{3}}$$

Jul 23-10:30 AM

If $f(x)$ is a Cont. Function on $[a,b]$, then the average of all values of $f(x)$ on $[a,b]$ is

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Find f_{ave} for $f(x) = x^2$ on $[0,1]$.



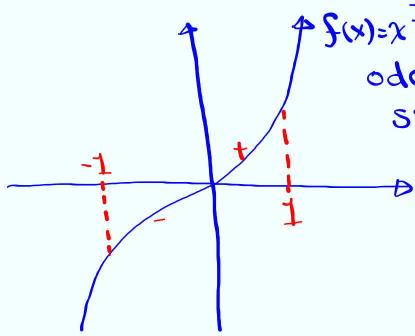
$$f_{ave} = \frac{1}{1-0}\int_0^1 x^2\,dx = \frac{1}{1}\cdot\frac{x^3}{3}\Big|_0^1 = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}(1^3-0^3) = \boxed{\frac{1}{3}}$$

Jul 23-10:37 AM

Find f_{ave} for $f(x) = x^3$ on $[\overset{a}{-1},\overset{b}{1}]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{1-(-1)}\int_{-1}^1 x^3\,dx$$

$$= \frac{1}{2}\cdot\frac{x^4}{4}\Big|_{-1}^1 = \frac{1}{8}x^4\Big|_{-1}^1 = \frac{1}{8}\left[1^4-(-1)^4\right]$$



$f(x) = x^3$

odd

sym → origin

$$= \frac{1}{8}[1-1] = \boxed{0}$$

Jul 23-10:43 AM

Find $\int 2x \cos x^2 \, dx$

$2x$ is the derivative of x^2.

Let $u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x \, dx$

$\int \cos u \, du$

$= \sin u + C$

$= \boxed{\sin x^2 + C}$

check

$\frac{d}{dx}\left[\sin x^2 + C\right] = \boxed{\cos x^2 \cdot 2x}$

Jul 23-10:50 AM